GAUSSIAN BEAM IMAGING WITH CYLINDRICAL OPTICS

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Summary

We review the propagation of Gaussian beams of radiation through asymmetric elements which focus in only one direction perpendicular to the axis of propagation: these are most commonly cylindrical lenses or mirrors. We develop formulas for Gaussian beam imaging with fixed total distance between input and output waists, and use them to derive simple expressions for the case where the input and output waists are located in the same plane. This situation is required for correcting the beam patterns of certain types of feedhorns to improve the symmetry of their radiation patterns, as well as for the illumination of asymmetric antennas. A cylindrical lens has been used to transform the symmetric beam of a 100 GHz scalar feedhorn into an elliptical pattern; the results are in good agreement with the imaging formulas and demonstrate the utility of this technique.

Introduction

Gaussian beams are widely used together with quasioptical systems at millimeter and submillimeter wavelengths. The propagation of a bundle of reasonably well-collimated radiation having a Gaussian field (and power) distribution perpendicular to its axis of propagation has been analyzed in detail and quite well understood for some time [(1), (2)]. This formulation of free space propagation is relatively straightforward, gives good agreement with measurements, and is thus a valuable design tool for design and analysis of many quasioptical components for which diffraction is a significant consideration [(3)]. One significant aspect of Gaussian beam propagation is that the sizes and curvatures of the beam in two orthogonal directions perpendicular to the axis of propagation are independent. Non-axisymmetric optical elements can thus be used to produce a beam having an elliptical rather than a circular cross section. A second use of such optical elements is to correct asymmetries in the radiation patterns of antennas. The patterns of certain types of feedhorns (such as smooth-walled conical horns) are largely Gaussian, but with significantly different beamwidths in the principal planes. The beam from this type of horn can be converted to a symmetric beam using cylindrical optics.

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Imaging with Fixed Waist Separation

A Gaussian beam has the electric field distribution given by

$$E(r)/E(o) = \exp[-(r/w)^2] \tag{1}$$

where r is the distance from the axis of propagation and the beam radius w is the beam size at a particular distance along the axis of propagation. The beam attains its minimum size at the beam waist, where the wavefronts are plane. The beam radius there is called the waist radius, w_o. Imaging of Gaussian beams is defined in terms of the size and location of the output waist produced by a focusing element operating on an input beam having specified waist radius and location relative to the focusing element. We denote the input waist radius by w_{o1} and the output waist radius by w_{o2}. The usual formulas which relate the input and output waists give the distances from the focusing element in terms of the input and output waist radii [(2)]. Alternatively, we can define the system magnification as the ratio of the output to the input waist radius

$$M = w_{o2} / w_{o1} \tag{2}$$

Let the total distance between the waists be given by d. In terms of the quantity

$$f_o = \pi w_{o1} w_{o2} / \lambda \tag{3}$$

the required focal length is

$$f = \frac{\pm[(M-1/M)^2 f_o^2 + d^2]^{1/2}(M+1/M) - 2d}{(M-1/M)^2} \tag{4a}$$

for M not equal to 1; the use of this equation is discussed further in [(4)]. It applies to the imaging of a symmetric beam or to a particular axis in an asymmetric system. The distances of

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the input waist (d_1) and output waist (d_2) from the lens are given by

$$d_1 = \frac{d - f(1-M^2)}{1 + M^2}, \quad d_2 = \frac{M^2 d + f(1-M^2)}{1 + M^2} \quad (4b)$$

Zero Waist Separation

In the case d=0, we see that

$$f = \frac{(M^2+1)}{(M^2-1)} f_o \qquad d_1 = -d_2 = f_o \qquad (5)$$

It is apparent that if we wish to produce an asymmetric Gaussian beam of given ellipticity, that the focal length of the cylindrical lens or mirror required and the distances from the waists are readily determined from Eq. 5.

Measurements

In order to illuminate an elliptical antenna, a Gaussian beam with approximately a three to one ratio of major to minor axes was required. The far-field beam divergence angle is inversely proportional to the beam waist radius. The most straightforward method of producing the pattern required is to use a cylindrical lens. For proper operation of the antenna, it is necessary that the beam waists in both planes perpendicular to the axis of propagation be coincident so that the zero waist separation situation discussed above is applicable. The geometry of the situation dictated the use of transmissive rather than reflective optics.

The symmetric Gaussian beam is produced by a scalar feedhorn having a waist radius 0.51 cm at a wavelength of 0.3 cm (f = 100 GHz). This type of antenna produces a highly symmetric beam which is nearly perfectly Gaussian in form [5]. To produce the elliptical beam we use a plano-convex cylindrical lens made of fused silica which is anti-reflection coated on both faces. The lens focal length is 10 cm. As can be verified from Eq. 5 the input and output waists are coincident at a distance 8.2 cm from the lens, with the desired ratio of waist radii. The measured radiation pattern of the feedhorn-lens combination is shown in Figure 1. The ellipticity is 3.0 rather than the value of 3.2 expected; this may be due to the truncation of the beam at the -11 dB level by the lens.

Conclusion

We have developed formulas for Gaussian beam imaging which are particularly useful for production of non-circular beams using asymmetric focusing elements. We have verified the predictions of the imaging formulas using a cylindrical lens-scalar feedhorn combination operating at a frequency of 100 GHz to produce an elliptical beam for illuminating a specially shaped antenna. Another application of this technique would be correcting the asymmetry of certain types of feedhorns, such as smooth-walled conical feedhorns [6] in order to improve the efficiency of illuminating symmetric antennas.

References

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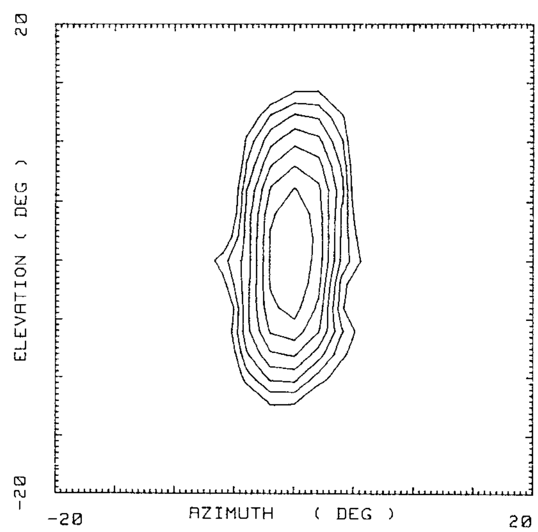

Fig. 1 Radiation pattern of scalar feedhorn and cylindrical lens. The innermost contour in each is 3 dB below the peak and the contour spacing is 3 dB.